Pergamon

www.elsevier.com/locate/jappmathmech

PII: S0021-8928(00)00125-8

J. Appl. Maths Mechs, Vol. 64, No. 6, pp. 945–950, 2000

0021-8928/00/$—see front matter

THE EXCITATION OF TSUNAMI BY LOW-FREQUENCY NON-STATIONARY POINT AND DISTRIBUTED CENTRES OF EXPANSION†

A. B. ZIMIN, V. A. KALINICHENKO,
S. Ya. SEKERZH-ZEN'KOVICH and I. K. SHINGAREVA

Moscow and Hermosillo (Mexico)

e-mail: shing@ipmnet.ru; seker@ipmnet.ru

(*Received* 8 *February* 2000)

The elevation of the free surface of a liquid relative to the bottom is calculated for a layer of incompressible heavy liquid with a thickness of 4 km located over an elastic half-space with a point or distributed centre of expansion acting on it at a depth of 40 km, the intensity of which undergoes stationary transient oscillations. The dependence of this elevation on the period of the source is investigated in the case when the period is greater than 2 minutes but less than 40 minutes. It is established that observable tsunamis are only excited by low frequency oscillations of the source.

It is usually assumed in theoretical investigations of the process involved in the excitation of a tsunami by seismic sources that these sources either act instantaneously [1, 2] or their intensity is a periodic function of time [3, 4]. A non-stationary centre of expansion was considered as a seismic source in [5], the power of which changes in the following manner over an infinite time interval: it slowly increases up to a certain maximum value and then executes a few oscillations after which it slowly decreases. The spectral function of the source was chosen such that an approximate solution of the problem could be obtained analytically and, in particular, it only contains harmonics with periods from 1 to 10 minutes. It was concluded that the strongest tsunami is excited by the longest period, 10 minute oscillations of the source. However, it is known from observations [6] that, in the case of strong tsunamis, the time interval between successive waves is usually is found, to be greater than 15 minutes.

The aim of this paper is to generalize the problem formulated in [5] in two directions: (1) to extend the spectrum of the source oscillations into the low-frequency domain and (2) to consider distributed sources. As in [5], it is necessary to ascertain which source parameters determine the greatest tsunamis.

The choice of a centre of expansion as the main source must be considered as an intermediate step. By constructing linear combinations of the partial derivatives of the resulting solution with respect to the spatial coordinates, it is possible to model the action of dipole sources. We note, however, that, in the model which is similar but has sources that are periodic in time, consideration of a zero-moment double dipole, which is used in seismology to model a real earthquake [6], did not lead to observable changes in the heights of the tsunami waves compared with the case of a centre of expansion.

1. PHYSICAL FORMULATION OF THE PROBLEM

Suppose a homogeneous elastic half-space $z < 0$ (the bottom) is covered with a layer of homogeneous heavy incompressible fluid (an ocean) of density ρ_0 and depth h_0 (Fig. 1). The fluid is bounded on top by a free surface and the pressure over this surface is assumed to be constant. The problem is investigated within the framework of plane deformation. A centre of expansion (a seismic source) is located in the elastic half-space at depth h and this source radiates axially symmetric, longitudinal waves which excite gravitational; waves in the fluid. The axis of the source (of a cylindrical wave) coincides with the Oy axis, which is perpendicular to the plane of the sketch.

Continuity of the vertical displacements of the fluid and the bottom is required at the interface of the elastic half-space and the fluid (on the bottom of the ocean) together with continuity of the normal stresses and the absence of tangential stresses (the fluid is ideal).

†*Prikl. Mat. Mekh.* Vol. 64, No. 6, pp. 983–988, 2000.

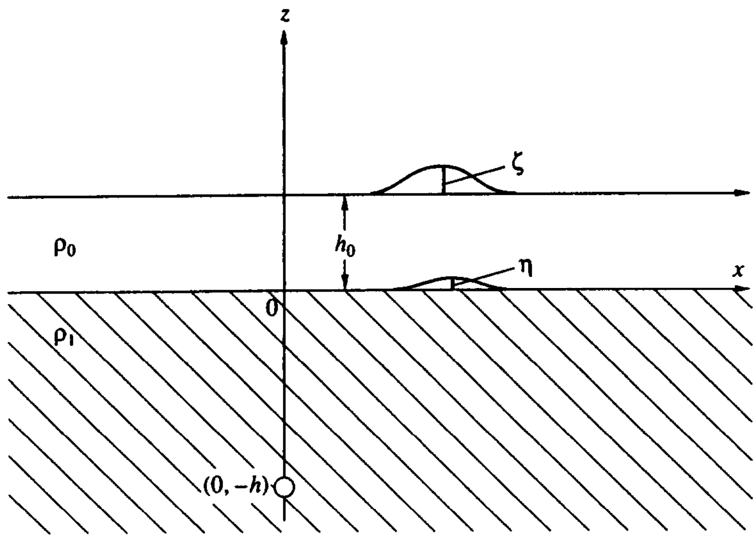

Fig. 1

It is assumed that gravity acts on the elastic medium but this action is approximately taken into account only in the boundary condition in the contact surface between this medium and the fluid. The displacements of the fluid particles and the elastic medium are assumed to be small, and the gravitational waves in the fluid are assumed to be long.

The problem is to ascertain under what conditions the seismic source excites relatively high gravitational waves in the fluid (a tsunami). More precisely, we shall assume that the waves are high if $\max(\zeta - \eta) \geqslant h^*$ where $\zeta(x, t)$ and $\eta(x, t)$ are the vertical displacements (in metres) of the free surface of the fluid and the bottom, and h^* is a certain chosen height.

2. THE POTENTIAL OF A NON-STATIONARY CENTRE OF EXPANSION

The scalar potential $\varphi_0(x, z, t)$ of a centre of expansion satisfies the equation

$$\partial^2\varphi_0 = a^2\Delta_2\varphi_0 + 4\pi a^2 q^2 \delta(x, z+h) f(t) \tag{2.1}$$

Here q is a constant which characterizes the power of the centre of expansion and has the dimension of length, a is the velocity of the longitudinal waves in the elastic medium, Δ_2 is the two-dimensional Laplace operator, δ is the δ-function and $f(t)$ is a function which determines the change in the power of the source with time.

We initially put

$$f(t) = \cos 2\pi t T \exp\left[-\frac{1}{K^2}\left(\frac{t}{T}\right)^2\right] \tag{2.2}$$

where K is a natural number and T is a certain positive quantity which has the dimension of time.

A graph of the function $f(t)$ for $K = 2$ and $T = 20$ min is shown in Fig. 2.

We see that, when $|t/T| > 3$, the function $f(t)$ is practically equal to zero. When $|t/T| > 3$, the function $f(t)$ describes oscillations with period T. Their amplitude initially increases up to unity (when $t = 0$) and then decreases. In this case, the number of oscillations with amplitudes greater than 0.3 is equal to five. Note that, in the general case, the number of such oscillations is equal to $K + 3$.

The Fourier transform of the function $f(t)$ has the form

$$F(\omega) = \sqrt{\frac{2}{\pi}}\int_0^\infty f(t)\cos\omega t\,dt = \frac{1}{\sqrt{2}} KT\exp(-\pi^2K^2)\exp\left[-\frac{1}{4}(KT\omega)^2\right]\mathrm{ch}(\pi K^2 T\omega) \tag{2.3}$$

We now choose, as $f(t)$, the function

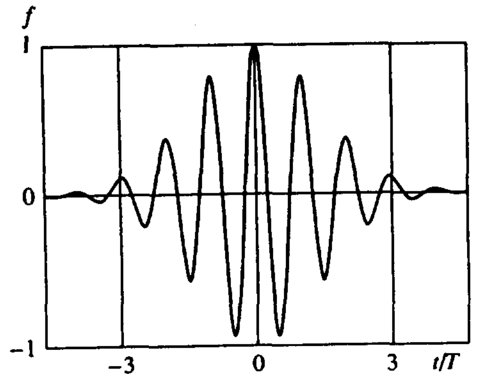

Fig. 2

$$f(t) = \frac{1}{\sqrt{\pi}} \int_{\omega_0}^{\omega_1} F(\omega)\cos(\omega t)d\omega \tag{2.4}$$

where ω_0 is the minimum and ω_1 is the maximum frequency in the spectrum of the source oscillations. We put $\omega_0 = 2\pi/T_{max}$, where $T_{max} = 42$ min. and $\omega_1 = 2\pi/T_0$, where $T_0 = 1$ min.

The graph of function (2.4) is practically the same as the graph of function (2.2) shown in Fig. 2.

The potential $\varphi_0(x, z, t)$ of the displacements of the source which satisfies Eq. (2.1) is

$$\varphi_0 = -i\sqrt{\frac{\pi}{2}}q^2 \int_{\omega_0}^{\omega_1} F(\omega)\left[H_0^{(2)}\left(\frac{\omega}{a}r\right)\exp(i\omega t) + H_0^{(2)}\left(-\frac{\omega}{a}r\right)\exp(-i\omega t)\right]d\omega \tag{2.5}$$

$$r = \sqrt{x^2 + (z+h)^2}$$

where $H_0^2(\cdot)$ is a zero-order Hankel function of the second kind and the function $F(\omega)$ is defined by formula (2.3).

3. APPROXIMATE FORMULA FOR THE ELEVATION OF THE SURFACE OF THE FLUID RELATIVE TO THE BOTTOM

The elevation of the free surface of the fluid relative to the elastic bottom, that is, $\zeta - \eta$, is determined as follows. Initially, formulae are derived for ζ and η in the form of repeating integrals with respect to the wave number k and the frequency ω using Fourier integral transformations with respect to time and with respect to the horizontal coordinate x. Then, using theorems on residues, the internal integrals with respect to k are reduced to sums of residues at the positive poles of the integrands which, in [4, 5], are called the Rayleigh pole" and the "tsunami pole", and, also, to integrals along the sides of the cuts in the plane of complex k. After this, the difference $\zeta - \eta$ is calculated. If the treatment is restricted to a domain which is remote from the epicentre of the seismic source, the integrals along the sides of the cuts and the residues at the "Rayleigh pole" can be omitted, As a result, the difference $\zeta - \eta$ is expressed in terms of the residue at the "tsunami pole" k_{ts} and can be represented as

$$\begin{gathered}\zeta(x,t) - \eta(x,t) \approx 8\pi\sqrt{\pi}\frac{q^2}{b}\frac{T}{T_{\max}^2}K\exp(-\pi^2K^2)\times \\ \times\int_1^{s_1} \Phi(s)\exp\left(-\frac{2\pi h}{bT_{\max}}s\sqrt{k_{ts}^2 - \alpha^2}\right)\sin\left[\frac{2\pi}{T_{\max}}\left(\frac{xk_{ts}}{b} - t\right)s\right]ds \\ \Phi(s) = s\exp\left[-\left(\frac{K_\pi Ts}{T_{\max}}\right)^2\right]\frac{2k^2-1}{\partial\Delta/\partial k}\bigg|_{k=k_{ts}} \mathrm{ch}\frac{2\pi^2K^2Ts}{T_{\max}} \\ \Delta(k,s) = (\gamma^2k^2 - 1)R(k) + \frac{gT_{\max}}{2\pi bs}\nu_\alpha\left(1 - \frac{\rho_1 - \rho_0}{\rho_1}\gamma^2k^2\right)\end{gathered} \tag{3.1}$$

$$R(k) = (2k^2 - 1)^2 - 4k^2\nu\nu_\alpha, \quad \nu = \sqrt{k^2 - 1}, \quad \nu_\alpha = \sqrt{k^2 - \alpha^2}$$

$$\gamma = \frac{\sqrt{gh_0}}{b}, \quad \alpha = \frac{b}{a}, \quad s_1 = \frac{T_{\max}}{T_0}$$

where k_{ts} is a real positive root of the equation

$$\Delta(k, s) = 0 \tag{3.2}$$

which has values close to the number 17 when $1 \leqslant s \leqslant s_1$. Note that the second positive root of Eq. (3.2) has values close to unity.

The elevation of the free surface of the fluid relative to the bottom in a domain remote from the epicentre of the seismic source was calculated using formula (3.1). The integration was carried out numerically and, moreover, the root k_{ts} of Eq. (3.2) was also calculated numerically at each integration step.

Note that, in [5], the approximate formulae for ζ and η were derived analytically with the additional assumption that $g/(\omega_0 b) \ll 1$, where b is the velocity of the transverse waves in the elastic bottom. This assumption requires that the period of the source should be less than 10 minutes. This assumption is not made in this paper.

4. RESULTS OF CALCULATIONS USING THE APPROXIMATE FORMULA

The difference $\zeta - \eta$ between the elevations of the free surface of the fluid and the elastic bottom was calculated using formula (3.1) with the following values of the main physical parameters of the problem:

$$\rho_0 = 10^3 \text{ kg/m}^3, \quad \rho_1 = 5 \times 10^3 \text{ kg/m}^3, \quad a = 5890 \text{ m/s}, \quad b = 3400 \text{ m/s}, \quad h_0 = 4000 \text{ m}$$

The depth h at which the source was located was taken to be 40 km. A change in h within the range 30–60 km only had a small effect on the form of the results. The point of observation was assumed to be remote from the epicentre of the source at a distance $x = 300$ km. In the case of an earthquake of magnitude 7, the source can then be approximately assumed to be a point source. The value of the quantity q in Eq. (2.5) was taken to be 300 m. In this case, the vertical displacement of the bottom, which is determined by the "Rayleigh pole" at a distance of 300 km from the epicentre of the source with $T = 20$ min, was of the order of 2 cm.

The period of the oscillations T (see formula (2.2)) was taken as the main variable source parameter. The magnitude of the parameter K only has a small effect on the qualitative form of the results and was taken as being equal to two.

Bearing in mind the analysis of the low-frequency source oscillations, the calculations were carried out for values of T in the range 5–40 min in steps of 5 min. It was found that the largest values of $\zeta - \eta$ are reached at $T = 20$ min. The corresponding graph of $(\zeta - \eta)/h^*$ against time t for $T = 20$ min is shown in Fig. 3. The quantity h^*, which has the dimension of length, is associated with the criterion according to which waves in a fluid are classified as tsunamis. Here, we have put $h^* = 0, 1$ m and it is assumed that the waves are classified as tsunamis if $\max(\zeta - \eta)/h^* > 1$. Hence, waves, in which the maximum elevation of the free surface of the fluid relative to the bottom exceeds 10 cm, are attributed to tsunamis.

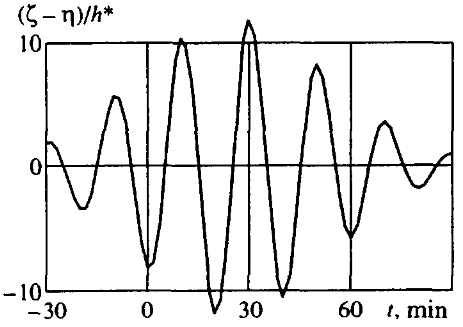

Fig. 3

We now compare Figs 2 and 3. We see that oscillations in the source intensity cause oscillations of the surface of the fluid of a similar form and of the same period $T = 20$ min. The maximum source intensity is reached when $t = 0$ (Fig. 2). The maximum height of the waves in the fluid at a distance of 300 km from the epicentre of the source in a tank with an absolutely solid bottom would be expected at a time $t = 25$ min since, according to the theory of long waves, the wave velocity, which is determined by the formula $\sqrt{gh_0}$, is equal to 0.2 km/s. According to Fig. 3, the maximum height of the waves is observed at a time $t = 30$ min, that is, almost a quarter of the period of the source oscillations later.

The results of calculations for other periods of the source intensity oscillations showed that the form of the oscillations of the fluid surface does not change at the point of observation. The maximum wave arrives at the point of observation approximately a quarter of a period later than might be expected according to hydrodynamic theory. The height of this wave decreases as the period deviates from 20 min. The results of calculations of the dependence of the greatest relative height of the wave $H = \max(\zeta - \eta)/h^*$ on the period T (in minutes) are presented below:

T	5	10	15	20	25	30	35	40
H	2.16	8.40	11.18	11.86	11.68	11.08	10.49	9.91

5. DISTRIBUTED SOURCES

We will first consider a vertically distributed centre of expansion, the potential φ_0^v of which satisfies the equation

$$\partial^2\varphi_0^v = a^2\Delta_2\varphi_0^v + 4\pi^2 a^2 q^2 f^v(z)\delta(x)f(t) \tag{5.1}$$

$$f^v(z) = \begin{cases} 1/(2dz), & \text{if} \quad -h-dz<z<-h+dz \\ 0, & \text{if} \quad |z+h|>dz \end{cases}$$

The quantity dz specifies half the vertical size of the source.

Using the δ-function, we represent $f^v(z)$ in the form

$$f^v(z) = \frac{1}{2dz}\int_{-h-dz}^{-h+dz}\delta(\sigma - z)d\sigma \tag{5.2}$$

Using the equality

$$\delta(z+h) = \delta(\sigma - z), \qquad \sigma = -h$$

and comparing Eqs (2.1) and (5.1), after taking account of expression (5.2), we conclude that the difference $\zeta - \eta$ in the case of a vertically distributed source can be calculated using a formula obtained from (3.1) after the following transformation. On the right-hand side of (3.1), it is necessary to replace h by $-\sigma$, integrate with respect to σ and divide by $2dz$. After these operations, we arrive at the conclusion that, in the case of a vertically distributed source, the integrand on the right-hand side of (3.1) has to be divided by $\omega_0 s b^{-1}\sqrt{k_{ts}^2 - \alpha^2}\, dz$ and multiplied by $\mathrm{sh}(\omega_0 s b^{-1}\sqrt{k_{ts}^2 - \alpha^2}\, dz)$.

In the case of a horizontally distributed source, the potential ψ_0^h of which satisfies the equation

$$\partial^2\varphi_0^h = a^2\Delta_2\varphi_0^h + 4\pi^2 a^2 q^2 f^h(x)\delta(z+h)f(t) \tag{5.3}$$

$$f^h(x) = \begin{cases} 1/(2dx), & \text{if} \quad -dx<x<dx \\ 0, & \text{if} \quad |x|>dx \end{cases}$$

the right-hand side of (3.1) has to be changed in the following way: the integrand has to be divided by $\omega_0 s b^{-1}\sqrt{k_{ts}^2 - \alpha^2}\, dx$ and multiplied by $\sin(\omega_0 s b^{-1}\sqrt{k_{ts}^2 - \alpha^2}\, dx)$.

The greatest relative wave height H in the fluid was calculated for a vertically distributed source for values of ρ_0, ρ_1, a, b, h_0, h. The period T of the source was chosen to be equal to 20 min, that is, the period of the wave of maximum height which is excited by a point source.

The calculations showed that, as half the vertical size of the source dz increases, H increases monotonically from 11.86 when $dz = 0$ to 12.43 when $dz = 20$ km.

Similar calculations for a horizontally distributed source led to the opposite conclusion, that is: an increase in half the horizontal size of the source dx leads to a monotonic decrease in the height from 11.86 when $dx = 0$ to 7.43 when $dx = 60$ km.

This research was supported financially by the Russian Foundation for Basic Research (97-05-64472).

REFERENCES

1. POD'YAPOL'SKII, G. S., Excitation of long gravitational wave in an ocean by a seismic source in the crust. *Izv. Akad. Nauk SSSR. Fizika Zemli*, 1968, 1, 7–24.
2. ALEKSEYEV, A. S. and GUSYAKOV, V. K., Numerical modelling of the excitation of tsunami waves and seismo-acoustic waves accompanying earthquakes in the ocean. In *Theory of Wave Diffraction and Propagation*, Vol. 2, *Proc. of the Sixth All-Union Symposium on Wave Diffraction and Propagation*, Tsakhkadzor, 1973. Vsesoyuh. Inst. Radiofizich. Izmerenii, Yerevan, 1973, 194–197.
3. ZVOLINSKII, N. V., The seismic mechanism of the excitation of tsunami waves, *Izv. Akad. Nauk SSR. Fizika Zemli*, 1986, **3**, 3–15.
4. ZVOLINSKII, V., NIKITIN, I.S. and SEKERZH-ZEN'KOVICH, S. Ya., Excitation of tsunami waves and Rayleigh waves by a harmonic centre of expansion. *Izv. Akad. Nauk SSSR. Fizika Zemli*, 1991, **2**, 34–44.
5. SEKERZH-ZEN'KOVICH, S. Ya., The problem of hydroelasticity when tsunami waves and Rayleigh waves are excited by a non-stationary centre of expansion of the pulse type. *Dokl. Ross. Akad. Nauk*, 1998, **363**, 338–340.
6. SOLOV'YEV, S. L. and GO, CH. N., *Catalogue of Tsunami on the Western Coast of the Pacific Ocean*, Nauka, Moscow, 1974.
7. SEKERZH-ZEN'KOVICH, S. Ya., ZAKHAROV, D. D., TIMOKHINA, A. O. and SHINGAREVA, I. K., Excitation of tsunami waves in a homogeneous ocean by seismic type sources in the Earth's crust. In *Interaction in the Lithosphere-Hydrosphere-Atmosphere System*. Collection of Papers Presented at the All-Russia Conference, Moscow, 1996, Vol. 2, Izd. Moskovsk. Gos. Univ. Fiz. Fak., Moscow, 1999, 233–240.

Translated by E.L.S.